

FABIO DELLA MARRA ^A

A FORECASTING PERFORMANCE
COMPARISON OF DYNAMIC
FACTOR MODELS BASED ON STATIC AND
DYNAMIC METHODS

^A Department of Economics, University of Parma, Parma - Italy

WORKING PAPERS
OF THE DEPARTMENT OF ECONOMICS

University of Parma - Italy
WP 2017-ME01
January 2017

Abstract

We present a comparison of the forecasting performances of three Dynamic Factor Models on a large monthly data panel of macroeconomic and financial time series for the UE economy. The first model relies on static principal-component and was introduced by Stock and Watson in [1], [2]. The second is based on generalized principal components and it was introduced by Forni, Hallin, Lippi and Reichlin in [3], [4]. The last model has been recently proposed by Forni, Hallin, Lippi and Zaffaroni in [5], [6]. The data panel is split into two parts: the calibration sample, from February 1986 to December 2000, is used to select the most performing specification for each class of models in a in-sample environment, and the proper sample, from January 2001 to November 2015, is used to compare the performances of the selected models in an out-of-sample environment. The methodological approach is analogous to [7], but also the size of the rolling window is empirically estimated in the calibration process to achieve more robustness. We find that, on the proper sample, the last model is the most performing for the Inflation. However, mixed evidencies appear over the proper sample for the Industrial Production.

Keywords: Macroeconomic Forecasting, Dynamic Factor Models, Time-domain methods, Frequency-domain methods

JEL subject classification: C0,C01, E01

1. Introduction

In this paper, a comparative analysis of the forecasting performance of three Large-Dimensional Dynamic Factor Models is presented. As a key feature, Dynamic Factor Models represent each variable in a dataset as the sum of two orthogonal terms: a *common component* χ_t , driven by a reduced (as compared to the number of series in the dataset) number of common factors, and an *idiosyncratic component* ξ_t , which represents measurement errors or local features. Among the different versions of the Dynamic Factor Models we selected:

- (i) *SW model*. This time-domain method was introduced in [1], [2]. The factors are estimated by computing static principal components of the variables in the dataset. Let y_{it} be the variable of the dataset to be forecasted at time t , its h -step-ahead prediction equation (also called *Diffusion Forecast Index*) is obtained by regressing y_{it+h} on the factors and on y_{it} itself. Lags of the factors and of y_{it} may be added.
- (ii) *FHLR model*. This frequency-domain method was proposed in [3], [4] and requires the computation of two steps. In a first step, the common component χ_t , the idiosyncratic component ξ_t and their covariances are estimated using a frequency-domain method introduced in [3] named *Dynamic Principal Component*. In the second step, the factors are estimated by computing Generalized Principal Components.
- (iii) *FHLZ model*. This frequency-domain method was proposed in [5], [6]. Here, the underlying assumption in (i) and (ii) that the common components span a finite-dimensional space as n tends to infinity is relaxed. The estimation of the parameters is much more complex though.

There exists some literature comparing the forecasting performances of SW and FHLR, but universal consensus still does not seem to have been reached. Theoretically, time-domain methods consider only relations among the variables at the same time, whereas frequency-domain methods exploit leaded and lagged relations among the variables. However time-domain methods require less parameters to be calibrated. Hence they are more robust to misspecification than frequency-domain methods. Empirically, in [8] Boivin and Ng found that SW generally outperforms FHLR on US macroeconomic data, whereas D’agostino and Giannone in [9] found no relevant differences in the performance of both methods on the whole sample (even though heterogeneity is found in subsamples). In [10] Schumacher found that FHLR outperforms SW on the prediction of the German GDP. The same conclusions are drawn in [11] over the forecasting of Dutch GDP. So far, a systematic comparison of the forecasting performances of SW, FHLR and FHLZ can be found only in [7]. Here, Forni et al. conducted a forecasting exercise on an US macroeconomic dataset, where they took an autoregressive process of order 4 as a benchmark. They showed that FHLZ outperforms SW, FHLR and the benchmark both for Industrial Production and Inflation during the Great Moderation. In the Great Recession, the forecasting performances of the Industrial Production change dramatically: all factor models are outperformed by the benchmark and SW and

FHLR outperform FHLZ. Hence, Forni et al. concluded that, due to its more dynamical structure, FHLZ tends to be the best performing method in “stationary period”, but it loses ground during regime changes. Also, they showed that FHLZ tends to be outperforming on nominal variables and FHLR on real variables.

In this paper, a large macroeconomic dataset consisting of 176 EU macroeconomic and financial time series observed at monthly frequency over the period from February 1986 to November 2015 is used to analyse the forecasting performance of these methods. To achieve stationarity, the series are deseasonalized and transformed. No treatment for outliers is applied.

As in [7], the EU dataset is split into two subsamples. The former, from February 1986 to December 2000, will be used to calibrate the models, i.e. to produce in-sample forecasts of the variables of the EU dataset for several specifications of SW, FHLR and FHLZ. Then, for each class of models, we selected the specification which shows the minimum mean square forecast error (MSFE). These models are then run and compared in the remaining sample, from January 2001 to November 2015.

The paper is structured as follows. In Section 2, the factor models are discussed in detail. In Section 3, the main features of the dataset are illustrated. In Section 4, the calibration process of the models is described. In Section 5, results are discussed and Section 6 concludes.

2. An overview on Dynamic Factor Models

Let $\{\mathbf{x}_t = (x_{1t} \dots x_{nt})' | t = 1, \dots, T\}$ be a n -dimensional vector of time series, which will be denoted as $\mathbf{x}_t$ for simplicity. $\mathbf{x}_t$ will be assumed to be weakly-stationary, purely non deterministic with zero mean and unit variance. Let us assume that the following decomposition holds:

$$(1) \quad \mathbf{x}_t = \chi_t + \xi_t$$

The process $\chi_t = (\chi_{1t} \dots \chi_{nt})'$ is called *common component*. It will be assumed that χ_{it} is stationary and that χ_{it} is costationary with χ_{jt} for all $i, j = 1, \dots, n$ such that $i \neq j$. The process ξ_t is called *idiosyncratic component*. It will be assumed that ξ_{it} is stationary and that ξ_{it} is costationary with ξ_{jt} for all $i, j = 1, \dots, n$ such that $i \neq j$. A distinction between *static* and *dynamic* factor models can be made according to the functional form selected for the common component. In static factor models, the common component can be modeled as $\chi_t = \mathbf{\Lambda} \mathbf{F}_t$ where $\mathbf{\Lambda} \in \mathbb{R}^{n \times r}$ is called the factor-loading matrix and $\mathbf{F}_t \in \mathbb{R}^r$ is the vector of the static factors at time

t , with $r \ll n$. In dynamic factor models, the common component takes into account also the lags of the factors. Hence, the common component can be modeled as $\chi_t = \mathbf{\Lambda}(L)\mathbf{u}_t$. Here L is the lag operator, $\mathbf{\Lambda}(L) \in \mathbb{R}^{n \times q}$ is called the factor-loading matrix and $\mathbf{u}_t \in \mathbb{R}^q$ is the vector of the dynamic factors at time t , with $q \ll n$. In [12], [13], it is shown that $\mathbf{u}_t$ is a orthonormal white noise process. Moreover, fixed a maximum lag order s for the matrix $\mathbf{\Lambda}(L)$, a dynamic factor model can be rewritten as a static factor model by stacking all of the factors with their lags in a single vector, i.e. by imposing $\mathbf{F}_t = (\mathbf{u}_t \dots \mathbf{u}_{t-s})$. More details can be found in [14]. This way, it holds that $r \ll q \ll n$. The three different dynamic factor models for estimating the factors are discussed in the following subsections.

2.1. The SW model

In [1], [2], Stock and Watson proposed a static factor model whose components are estimated by means of static principal component. Let $\hat{\mathbf{\Gamma}} = T^{-1} \sum_{t=1}^T \mathbf{x}_t \mathbf{x}_t'$ be the sample covariance matrix of $\mathbf{x}_t$. By computing the eigenvalues of $\hat{\mathbf{\Gamma}}$ and stacking them into the matrix $\mathbf{P} = (\mathbf{P}_1 \dots \mathbf{P}_r)'$, with $\mathbf{P}_i$ the eigenvector corresponding to the i -th largest eigenvalue, we can compute the factors $\hat{\mathbf{F}}_t = \mathbf{P}'\mathbf{x}_t$. The h -step ahead SW forecasting equation, also called *Diffusion Forecast Index*, can be computed by regressing x_{it+h} on the factors $\hat{\mathbf{F}}_t$ and x_{it} . Lags of both $\hat{\mathbf{F}}_t$ and x_{it} may be added. Hence, the Diffusion Forecast Index can be modeled as

$$(2) \quad x_{it+h|t} = \mathbf{a}_i(L)\hat{\mathbf{F}}_t + b_i(L)x_{it}$$

where $\mathbf{a}_i(L)$ is a $n \times r$ vector of polynomials of degree α_i and $b_i(L)$ is a scalar polynomial of degree β_i . Both $\mathbf{a}_i(L)$ and $b_i(L)$ can be estimated by OLS.

2.2. The FHLR model

This model was proposed by Forni et al. in [3], [4]. It is articulated in two steps:

- (i) *Estimation of the common and idiosyncratic component*: let $\hat{\mathbf{\Gamma}}(k) = T^{-1} \sum_{t=1}^T \mathbf{x}_t \mathbf{x}_{t-k}'$ be the sample autocovariance of $\mathbf{x}_t$ at lag k . In order to consistently estimate the spectral density matrix of $\mathbf{x}_t$, we can compute this estimator $\hat{\mathbf{\Sigma}}(\theta) = \sum_{k=-d}^d w_k \hat{\mathbf{\Gamma}}(k) e^{-ik\theta}$ with w_k being the weights of a kernel function. By computing the spectral decomposition of $\hat{\mathbf{\Sigma}}(\theta)$ for all θ , the spectral density

matrix of the common component can be reconstructed by computing $\hat{\Sigma}_\chi(\theta) = \hat{\mathbf{P}}(\theta)\mathbf{\Lambda}(\theta)\hat{\mathbf{P}}^*(\theta)$, where $\mathbf{\Lambda}(\theta)$ denotes the diagonal matrix whose entries are the q largest eigenvalues of $\hat{\Sigma}(\theta)$ and $\hat{\mathbf{P}}(\theta) = (\hat{\mathbf{P}}_1(\theta) \dots \hat{\mathbf{P}}_q(\theta))$ the matrix whose columns are the corresponding eigenvectors. The spectral density matrix of the idiosyncratic component can be reconstructed by differencing $\hat{\Sigma}_\xi(\theta) = \hat{\Sigma}(\theta) - \hat{\Sigma}_\chi(\theta)$. Autocovariances at lag k of the common and the idiosyncratic component can be obtained by computing the inverse Fourier transform of their estimated spectral density matrix.

- (ii) *Estimation of the factors*: now, the estimated covariance matrix of the common component and of the idiosyncratic component are used to solve the generalized principal components problem:

$$(3) \quad \hat{\Sigma}_\xi^0 \mathbf{P} = \hat{\Sigma}_\chi^0 \mathbf{P} \mathbf{D}$$

s.t. $\mathbf{P}' \hat{\Sigma}_\chi^0 \mathbf{P} = \mathbf{I}$, where $\mathbf{D}$ is a diagonal matrix whose entries are the r largest eigenvalues of the pair $(\hat{\Sigma}_\xi^0, \hat{\Sigma}_\chi^0)$ and $\mathbf{P}$ is the matrix containing the corresponding eigenvectors. The first r factors are defined as $\hat{\mathbf{F}}_t = \mathbf{P}' \mathbf{x}_t$

By means of the projections:

$$(4) \quad \begin{aligned} \hat{\chi}_{it+h|t} &= Proj[\chi_{it+h} | \hat{\mathbf{F}}_t] \\ \hat{\xi}_{it+h|t} &= Proj[\xi_{it+h} | \mathbf{x}_{it} \dots \mathbf{x}_{it-p}] \end{aligned}$$

the h -step ahead FHLR forecasting equation can be finally derived as

$$(5) \quad \chi_{it+h|t} = \hat{\chi}_{it+h|t} + \hat{\xi}_{it+h|t}$$

2.3. The FHLZ model

This model was proposed by Forni et al. in [5], [6]. Differently from the previous models, here the assumption that the common component spans a finite-dimensional space is relaxed. This model is articulated in the steps listed below:

- (i) *Estimation of the spectral density matrix* $\hat{\Sigma}^x(\theta)$ of $\mathbf{x}_t$: the spectral density matrix $\hat{\Sigma}^x(\theta)$ can be estimated as $\hat{\Sigma}^x(\theta) = \frac{1}{2\pi} \sum_{k=-R}^R \omega_k \hat{\mathbf{\Gamma}}(k) e^{-ik\theta}$ with ω_k representing the weights of a kernel function.

- (ii) *Estimation of the spectral density matrix $\hat{\Sigma}^x(\theta)$ of χ_t* : it is obtained by computing the dynamic principal components of $\hat{\Sigma}^x(\theta)$ and then by selecting its q principal components which are associated to the largest eigenvalues. For more details, see [3], [5].
- (iii) *Estimation of the autocovariance matrices $\hat{\Gamma}_k^x$ of χ_t* : The autocovariances of χ_t are estimated by means of the Wiener-Khinchin-Einstein theorem.
- (iv) *Estimation of the VAR matrices $\hat{\mathbf{A}}^k(L)$* : under general assumptions, the common component admits a unique blockwise autoregressive representation of the form:

$$(6) \quad \begin{bmatrix} \mathbf{A}^1(L) & 0 & \dots & 0 & \dots \\ 0 & \mathbf{A}^2(L) & \dots & 0 & \dots \\ \vdots & \vdots & \ddots & & \\ 0 & 0 & \dots & \mathbf{A}^k(L) & \dots \\ \vdots & & & & \ddots \end{bmatrix} \chi_t = \begin{bmatrix} \mathbf{R}^1 \\ \mathbf{R}^2 \\ \vdots \\ \mathbf{R}^k \\ \vdots \end{bmatrix} u_t$$

where $\mathbf{A}^k(L) \in \mathbb{R}^{(q+1) \times (q+1)}$ is a polynomial matrix of finite degree and $\mathbf{R}^k \in \mathbb{R}^{(q+1) \times q}$. To estimate the VAR matrices $\mathbf{A}^k(L)$, the covariances $\hat{\Sigma}^x(\theta)$ are employed.

- (v) *Estimation of the matrices $\mathbf{R}^k$ and the shock $\mathbf{u}_t$* : these estimates can be recovered by applying standard principal components to the process $\mathbf{A}(L)\mathbf{x}_t$.

By inverting equation (6) it follows that: $\chi_t = [\mathbf{A}(L)^{-1}]\mathbf{R}\mathbf{u}_t = \mathbf{C}(L)\mathbf{u}_t = \mathbf{C}_0\mathbf{u}_t + \mathbf{C}_1\mathbf{u}_{t-1} + \dots$ where $\hat{\chi}_t \in \mathbb{R}^n$ and $\hat{\mathbf{A}}(L), \hat{\mathbf{R}}, \hat{\mathbf{W}}(L) \in \mathbb{R}^{n \times n}$.

The h -step ahead FHLZ forecasting equation is reported below:

$$(7) \quad \chi_{it+h|t} = \mathbf{C}_h\mathbf{u}_t + \mathbf{C}_{h+1}\mathbf{u}_{t-1} + \dots$$

3. Description of the dataset

In this empirical application, a large macroeconomic dataset consisting of 176 EU macroeconomic and financial time series observed at monthly frequency is employed. This dataset contains real variables (import/export price indexes, employment, Industrial Production) and nominal variables (money aggregates, consumer price indexes, wages), asset prices (stock prices and exchange rates) and surveys. Further details can be found in the Appendix. To achieve stationarity, several series are deseasonalized and transformed. No treatment for outliers is applied. In addition to SW, FHLR,

FHLZ, the forecasts of an autoregressive process (AR) of order 4 are computed. The dataset is divided in two parts: a *calibration sample*, ranging from February 1986 to December 2000, which will be employed to select the most performing specification of each model, and a *proper sample*, ranging from January 2001 to November 2015, which will be employed to compare the selected specifications of each model. As in [2], [9], to assess the forecasting performances, the variables which are taken into account are the level of the logarithm of the Industrial Production (IP) and the yearly change of the logarithm of the Consumer Price Index (CPI). Forecasts are computed h -months ahead, with $h \in \{1, 3, 6, 12, 24\}$. For each methods, we employed a rolling-window scheme $[t - l, t]$, whose size l will be determined in the calibration sample. To assess the forecasting performance of each model, the mean-square forecast error (MSFE) is employed as a metric:

$$(8) \quad MSFE^h(i) = \frac{1}{(T_{end} - h) - T_{begin} + 1} \sum_{k=T_{begin}}^{T_{end}-h} SFE_k^h(i)$$

where T_{begin}, T_{end} stand, respectively, for the first and the last date in the dataset and $i \in \{SW, FHLR, FHLZ, AR\}$. SFE^h stands for h -step ahead squared forecast error and is defined as $SFE^h(i) = (y_{t+h|T}^i - y_{t+h})^2$, where $y_{t+h|T}^i$ is the forecasted value at horizon h of the variable y_t by the method i and y_{t+h} is its real value.

4. Calibration

The calibration procedure is basically the same as in [7], but is more robust since the size of the rolling window is taken into account as a parameter to be tuned. The calibration sample, ranging from February 1986 to December 2000, will be used to calibrate the methods SW, FHLR, FHLZ. Namely, this portion of the dataset will be used to select the best performing specifications of each class of models. To compare the performances of two different methods, say α, β , at a certain horizon h , the ratio mean-square forecast error (RMSFE) will be computed. Such metric is defined as

$$(9) \quad RMSFE^h(\alpha, \beta) = \frac{MSFE^h(\alpha)}{MSFE^h(\beta)}$$

4.1. Calibration of SW

To produce forecastings by means of equation (2), the following parameters must be calibrated:

- (i) *the number of static factors r* : ranging from 1 to 10. Also, a comparison with Bai & Ng criterium (BN) with maximum 12 factors has been made.
- (ii) *the degree α of $\mathbf{a}(L)$* : ranging from 1 to 10.
- (iii) *the degree β of $\mathbf{b}(L)$* : ranging from 0 to 10.
- (iv) *the size l of the rolling window*: ranging from 5 to 12 years.

By selecting the values of the parameters which guarantee the lowest mean RMSFE, the chosen configuration for the IP is the following:

$$(10) \quad (r, \alpha, \beta, l) = (4, 1, 0, 7)$$

Instead, the chosen configuration for the CPI is the following:

$$(11) \quad (r, \alpha, \beta, l) = (4, 1, 9, 12)$$

4.2. Calibration of FHLR

To produce forecastings by means of equation (5), the following parameters must be calibrated:

- (i) *the number of static factors r* : ranging from 1 to 10. Also, a comparison with Bai & Ng criterium (BN) with maximum 12 factors has been carried out.
- (ii) *the number of dynamic factors q* : ranging from 0 to 10. Also, a comparison with Hallin-Liska criterium (HL) with maximum 12 factors has been carried out.
- (iii) *the type of kernel k* : ranging in the set {Triangular, Rectangular, Parzen, Gaussian, Exponential, Cosine, Tukey, Hann}.
- (iv) *The lag window d for spectral density estimation*: ranging in the set {25, 35, 40}.
- (v) *the size l of the rolling window*: ranging from 5 to 12 years.

By selecting the values of the parameters which guarantee the lowest mean RMSFE, the chosen configuration for the IP is the following:

$$(12) \quad (r, q, k, d, l) = (4, 3, Tukey, 25, 12)$$

Instead, the chosen configuration for the CPI is the following:

$$(13) \quad (r, q, k, d, l) = (1, 3, Cosine, 25, 7)$$

4.3. Calibration of FHLZ

To produce forecastings by means of equation (7), the following parameters must be calibrated:

- (i) *the number of dynamic factors q* : ranging from 1 to 5. Also, a comparison with Hallin-Liska criterium has been carried out.
- (ii) *the type of kernel k* : ranging in the set {Triangular, Rectangular, Parzen, Gaussian, Exponential, Cosine, Tukey, Hann}.
- (iii) *the lag window d for spectral density estimation*: ranging in the set {25, 35, 40}.
- (iv) *the maximum lag ml for the matrix $\mathbf{A}^k(L)$* : ranging from 1 to 5.
- (v) *the size l of the rolling window*: ranging from 5 to 12 years.

By selecting the values of the parameters which guarantee the lowest mean RMSFE, the chosen configuration for the IP is the following:

$$(14) \quad (r, k, d, ml, l) = (4, \text{Parzen}, 25, 4, 12)$$

Instead, the chosen configuration for the CPI is the following:

$$(15) \quad (r, k, d, ml, l) = (2, \text{Rectangular}, 25, 1, 7)$$

5. Results on the proper sample

5.1. Prediction of the Industrial Production and the Inflation

Now, the forecasting performances of the three dynamic factor models over the IP and CPI are compared on the proper sample, which starts on January 2001 and ends on November 2015. To forecast the IP, we changed the size of the rolling window employed in SW from 7 to 12 years. Instead, to forecast the CPI, we changed the size of the rolling window employed in SW from 12 to 7 years. Hence, to forecast the IP and the CPI, all dynamic factor models employ a rolling window of the same length. The common benchmark for the factor models is the autoregressive process (AR) of order four. In table 1, the average RMSFEs (relative to the AR) of the selected dynamic factor models are reported for the IP and CPI on the whole proper sample. However, as reported by CEPR, during the proper sample, the european economy faces two crisis periods: the first starts on May 2008 and ends on January 2009. The second starts on September 2011 and ends on March 2013. Hence, it is reasonable to assess whether the relative forecasting performances of the three dynamic factor models present a relevant

change during the crisis periods. In table 2, the average RMSFEs (relative to the AR) of the three dynamic factor models are reported for the IP and the CPI from January 2001 to April 2008 (i.e. before the first crisis on the proper sample). In table 3, the average RMSFEs (relative to the AR) of the three dynamic factor models are reported for the IP and the CPI from January 2001 to August 2011 (i.e. before the second crisis on the evaluation sample). One, two or three asterisks indicate that the null of equal performance of the three factor models relative to AR is rejected at, respectively, the 1%, 5%, 10% significance by the Giacomini-White Test. One, two or three daggers indicate that the null of equal performance of FHLR, FHLZ relative to SW is rejected at, respectively, the 1%, 5%, 10% significance by the Giacomini-White Test. For further details about Giacomini-White Test, see [15].

Table 1. RMSFEs on the whole sample: IP on the left, CPI on the right.

$h/model$	IP			CPI		
	$FHLZ$	$FHLR$	SW	$FHLZ$	$FHLR$	SW
1	0.97	0.95	0.92	0.95	1.01	1.17
3	0.84	0.82	0.82	0.88***	0.95	1.08
6	0.66	0.66	0.68	0.85	0.94	1.04
12	0.80	0.81	0.79	0.85	0.87	1.18
24	0.94	0.94	0.92	1.02	1.02	1.47

Table 2. RMSFEs from January 2001 to April 2008: IP on the left, CPI on the right.

$h/model$	IP			CPI		
	$FHLZ$	$FHLR$	SW	$FHLZ$	$FHLR$	SW
1	1.15	1.20	1.24	0.96	0.98	0.99***
3	0.80	0.82	0.79	1.05	1.07	1.14
6	0.92	1.03	1.15	1.14**†	1.17**††	1.32*
12	0.91	1.03	1.03	1.00***†	1.04**††	1.32*
24	0.85	0.84*	0.86*	0.79	0.82	1.09

The relative performances of all methods tend to improve especially after the first crisis at horizons $h \in \{1, 3, 6, 12\}$. This holds both for IP and CPI. This is also illustrated in figure 1 (on the left), in which the graphs of the cumulated sums of the square forecast error at $h = 6$ for the IP are reported. The corresponding plots are similar for different values of $h \neq 24$ and are not reported here. The shaded areas correspond to the two crisis periods in the proper sample, according to CEPR. A relevant jump can be observed during the first crisis for all methods, including the benchmark, followed by a period of permanent flatness. However, after the first crisis,

A forecasting performance comparison of dynamic factor models based on static and dynamic methods

Table 3. RMSFEs January 2001 to August 2011: IP on the left, CPI on the right.

$h/model$	IP			CPI		
	$FHLZ$	$FHLR$	SW	$FHLZ$	$FHLR$	SW
1	0.95	0.87	0.82	0.97	1.06	1.24
3	0.84	0.77	0.78	0.90	0.97	1.08
6	0.65	0.65	0.66	0.85	0.92	0.97
12	0.79	0.80	0.76	0.85	0.87	1.11
24	0.98	0.98	0.93	1.24	1.23	1.82

the sum of the cumulative forecast errors of AR increases substantially in comparison with those of the dynamic factor models (e.g., on October 2009, the sum of the cumulative forecast errors of AR increases, on average, more than 50% in comparison with those of the dynamic factor models). Similar results are obtained for the CPI and are not reported here.

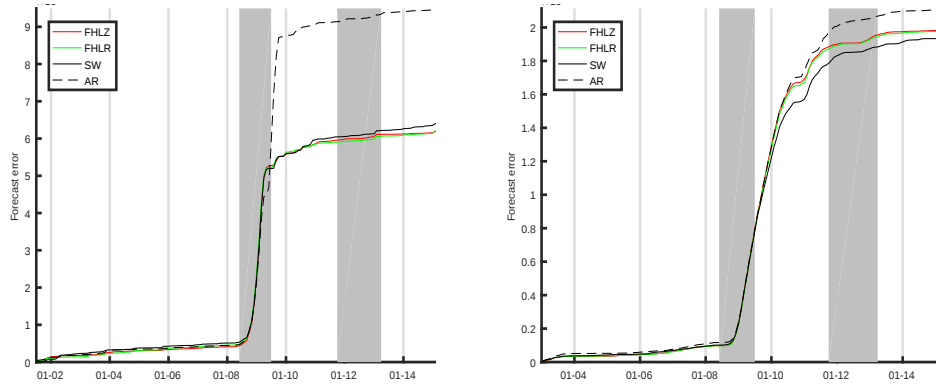


Figure 1. Cumulative sum of the square forecast error at $h = 6$ for the IP (on the left). Cumulative sum of the square forecast error at $h = 24$ for the IP (on the right).

Instead, at horizon $h = 24$, the relative performances of all methods tend to worsen after the first crisis. This is also illustrated in figure 1 (on the right), in which the graph of the cumulated sums of the square forecast error for the IP are reported. In this plot, a jump during the first crisis can be observed whose slope is less remarked than in the left-side plot. This behaviour seems to persist until the beginning of the second crisis, after which a period of permanent flatness arises. However, no substantial increase in the sum of the cumulative forecast errors of AR appears in comparison with the dynamic factor models. Similar results are obtained for the CPI and are not reported here.

As in [7], to assess the forecasting performance of each couple of methods locally, each time series of the dataset is smoothed by a centered moving

average of length $m = 61$ (with coefficients equal to $1/m$) and then the Fluctuation test ([16]) is run, at 5% significance level. The results for the IP at horizons $h \in \{6, 12, 24\}$ are reported in Figure 2. All methods outperform AR significantly from the first crisis on. At horizon $h = 6$, FHLR and FHLZ outperforms SW on average on the whole sample, except during the first crisis. Instead, at horizons $h \in \{12, 24\}$, SW outperforms FHLR and FHLZ from the first crisis on. FHLR tends to outperform FHLZ in the two crisis periods. Outside the crisis periods, FHLZ tends, on average, to outperform FHLR at horizons $h \in \{6, 12\}$. Instead, at horizon $h = 24$, FHLR outperforms FHLZ from the first crisis on.

The results for the CPI at horizons $h \in \{6, 12, 24\}$ are reported in Figure 3. FHLR and FHLZ tend to outperform SW at all horizons, except FHLR at horizon $h = 6$ during the former crisis.

FHLR and FHLZ tend to outperform SW at all horizons, except FHLR at horizon $h = 6$ during the first crisis. FHLR and FHLZ outperform AR at horizons $h \in \{6, 12\}$. At horizon $h = 24$, AR outperforms FHLR and FHLZ from the first crisis on. SW outperforms AR during the two crisis periods at horizons $h \in \{6, 12\}$. At horizons $h \in \{12, 24\}$, AR outperforms SW from the second crisis on. At all horizons, FHLZ outperforms FHLR during the first crisis. At horizons $h \in \{6, 12\}$, this behaviour seems to be permanent. Instead, at horizon $h = 24$, FHLR outperforms FHLZ from the second crisis on.

5.2. Prediction of the dataset

As in [7], this exercise has been extended to the other variables in the dataset. In table 4, we report the mean RMSFEs of each class of real (the first three) and nominal (the others) time series, taking AR as a benchmark. Similar results are obtained for the median and not reported here. The best performances are given in bold. We have excluded the categories “Demand”, “Money” and “Exchange Rates”, since AR outperformed all factor models at any horizon.

AR outperforms all methods at horizon $h = 1$ for all categories, except for Interest Rates. Also, AR generally outperforms the dynamic factor models at shorter horizons (within 6 months) and it is the best methods also at $h = 12$ for Unemployment and Wages. As to the factor models, FHLZ seem to be the most accurate methods for real variables. Instead, no method seems to be systematically the most performing in forecasting the nominal variables. In any case, dynamic methods seem to produce more precise forecasts than SW both on nominal and real variables. In table 5, the distribution of the RMSE of the dynamic models for each categories

A forecasting performance comparison of dynamic factor models based on static and dynamic methods

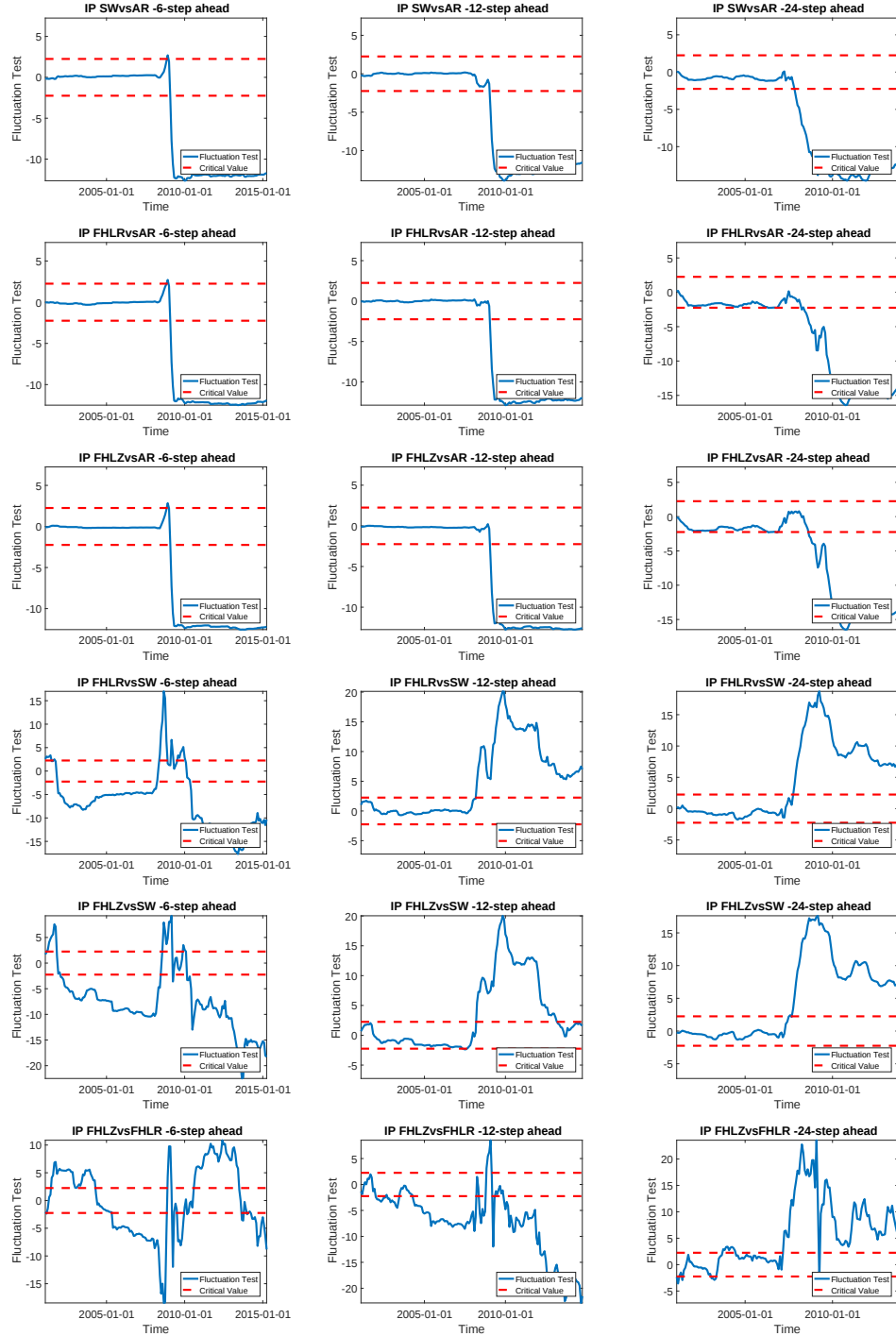


Figure 2. Fluctuation test for the IP.

Fabio Della Marra

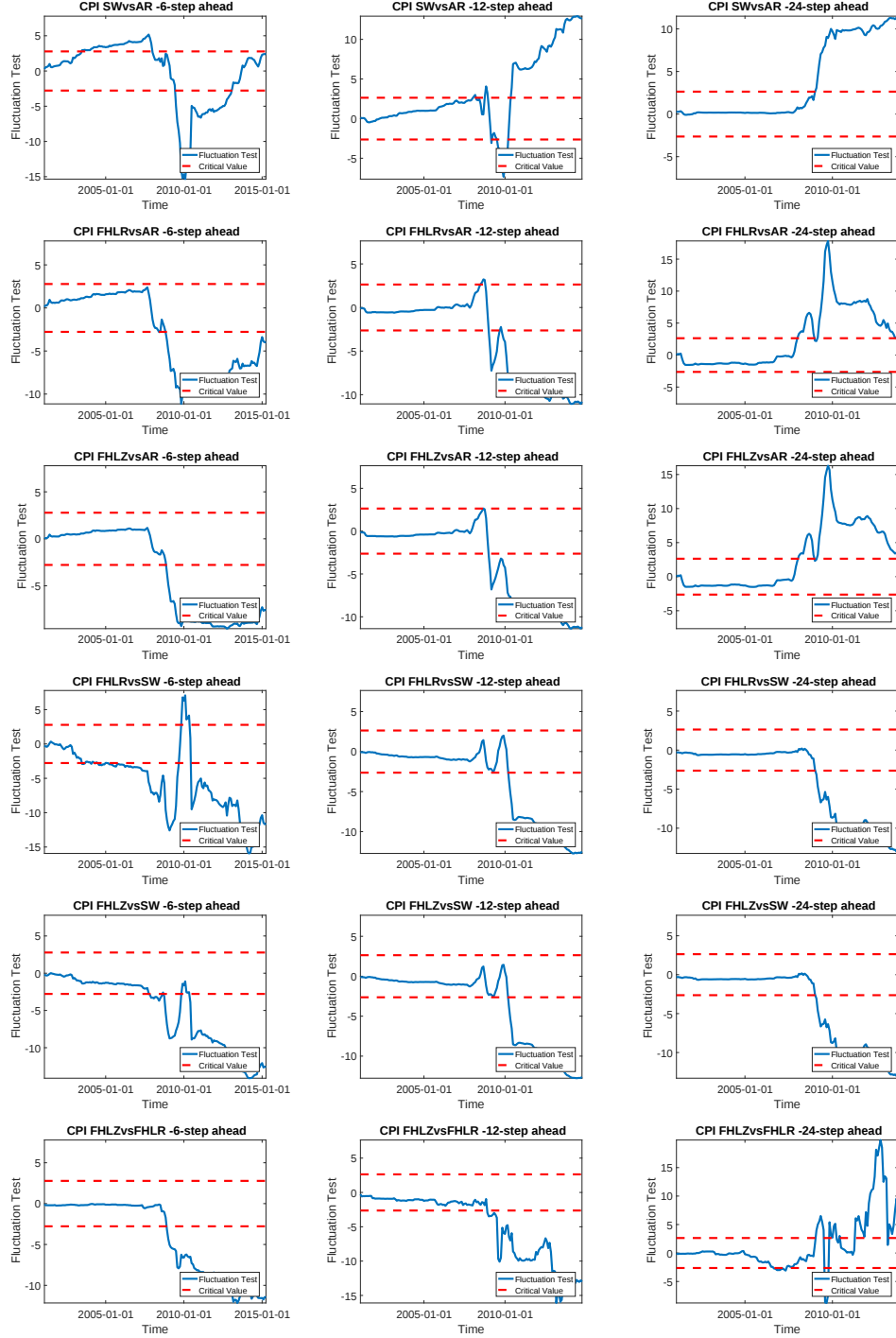


Figure 3. Fluctuation test for the CPI.

A forecasting performance comparison of dynamic factor models based on static and dynamic methods

Table 4. Mean RMSFE for category.

SW					
	$h = 1$	$h = 3$	$h = 6$	$h = 12$	$h = 24$
Import-Export	1.20	0.94	1.00	0.99	1.04
Unemployment	1.28	1.17	1.16	1.10	1.00
Industrial Production	1.12	1.01	0.93	0.92	0.96
Prices	1.01	0.97	0.96	1.01	1.16
Wages	1.19	1.28	1.39	1.33	1.31
Surveys	1.12	1.09	1.06	0.94	0.94
Interest Rates	1.06	1.11	1.08	1.06	1.00
Stock Prices	1.12	1.10	1.07	1.01	1.03

FHLR					
	$h = 1$	$h = 3$	$h = 6$	$h = 12$	$h = 24$
Import-Export	1.18	0.93	0.97	0.99	1.00
Unemployment	1.29	1.17	1.15	1.06	0.99
Industrial Production	1.14	1.00	0.93	0.94	0.98
Prices	1.19	1.17	1.11	1.04	1.11
Wages	1.07	1.09	1.14	1.02	0.98
Surveys	1.14	1.12	1.04	1.01	1.09
Interest Rates	0.94	0.93	0.91	0.89	0.84
Stock Prices	1.01	1.01	0.99	1.01	0.99

FHLZ					
	$h = 1$	$h = 3$	$h = 6$	$h = 12$	$h = 24$
Import-Export	1.16	0.97	0.96	0.97	1.00
Unemployment	1.22	1.09	1.07	1.02	0.97
Industrial Production	1.11	0.99	0.91	0.92	0.98
Prices	1.11	1.11	1.10	1.10	0.92
Wages	1.36	1.49	1.55	1.48	1.57
Surveys	1.06	1.09	1.06	1.09	0.99
Interest Rates	0.78	0.73	0.80	0.91	1.08
Stock Prices	1.13	1.14	1.13	1.22	1.29

are reported. The configuration of the parameters is the one chosen in the calibration process for the forecasting of the IP. Similar results are obtained for the configurations adopted for the CPI and are not reported here.

FHLZ is the only method which improves for at least half of the series. More precisely, FHLZ is as accurate as AR till half of the series at horizon $h = 1$. At the other horizons, it is as accurate as AR till the 75-th percentile. SW is outperformed by frequency domain methods at all horizons and at

Table 5. Distribution of the RMSFE.

SW					
Percentile	0.05	0.25	0.50	0.75	0.95
$h = 1$	0.89	1.02	1.08	1.17	1.53
$h = 3$	0.83	0.97	1.05	1.11	1.41
$h = 6$	0.81	0.96	1.03	1.13	1.39
$h = 12$	0.80	0.92	0.99	1.10	1.38
$h = 24$	0.82	0.94	1.03	1.14	1.39

FHLR					
Percentile	0.05	0.25	0.50	0.75	0.95
$h = 1$	0.91	0.98	1.04	1.14	1.48
$h = 3$	0.83	0.94	1.02	1.07	1.30
$h = 6$	0.82	0.95	1.01	1.07	1.30
$h = 12$	0.85	0.96	1.01	1.05	1.25
$h = 24$	0.88	0.96	1.00	1.05	1.19

FHLZ					
Percentile	0.05	0.25	0.50	0.75	0.95
$h = 1$	0.91	0.98	1.02	1.10	1.43
$h = 3$	0.85	0.95	0.99	1.07	1.28
$h = 6$	0.80	0.94	0.97	1.03	1.23
$h = 12$	0.83	0.93	0.97	1.01	1.17
$h = 24$	0.86	0.95	0.98	1.04	1.16

almost all percentiles. Within frequency domain methods, FHLZ performs the best at 5-th and 95-th percentile.

6. Conclusions

In this paper, the forecasting performances of SW, FHLR and FHLZ are compared on a EU macroeconomic dataset which spans from February 1986 to November 2015. The EU dataset is split into two parts: the former is called *calibration set* and is used to tune the parameters of the dynamic factor models. The latter is called *proper sample* and is used to compare the forecasting performances of the three dynamic factor models. In the proper sample, all methods outperform AR after the first crisis at horizons $h \neq 24$. Instead, at horizon $h = 24$, all methods tend to lose ground against AR after the first crisis. Moreover, FHLZ generally outperforms all methods on the prediction of the CPI. Instead, no method seems to outperform in

A forecasting performance comparison of dynamic factor models based on static and dynamic methods

forecasting the IP, but all dynamic factor models outperform the benchmark AR. In the forecasting the whole dataset exercise, FHLZ is the most performing method in predicting real variables, whereas no significant evidences appear on the prediction of nominal variables. However, apart from Interest Rates, all methods seem to perform poorly in comparison with AR.

Acknowledgements

The author would like to express his gratitude to Donatella Albano, Luca Consolini, Paolo Federici, Alessandro Giovannelli, Fabrizio Laurini, Marco Lippi, Aurelio Piazzzi, Simona Sanfelici and Stefano Soccorsi for their support and comments on an earlier version of the manuscript, although any errors are his own.

Appendix: Dataset and transformation

In table A.1, the series which compose the dataset are reported. T code identifies the tranformation (further details are given below) and Des identifies the deseasonalized flag (1 if the series is deseasonalized, 0 otherwise).

Table A1: List of the time series.

	Name	Long Desc.	Tcode	Des
1	BDM1....A	BD MONEY SUPPLY - M1 - CURA	6	1
2	BDM2C...B	BD MONEY SUPPLY - M2 CURA	6	0
3	BDM3C...B	MONEY SUPPLY - M3 - CURA	6	0
4	FRM1....A	FR MONEY SUPPLY - M1 - CURN	6	1
5	FRM2....A	FR MONEY SUPPLY - M2 - CURN	6	1
6	FRM3....A	FR MONEY SUPPLY - M3 - CURN	6	1
7	ITM1....A	IT MONEY SUPPLY: M1 - CURN	6	1
8	ITM2....A	IT MONEY SUPPLY: M2 - CURN	6	1
9	ITM3....A	MONEY SUPPLY: M3 - CURN	6	1
10	NLM1....A	NL MONEY SUPPLY - M1 - CURN	6	1
11	NLM2....A	NL MONEY SUPPLY - M2 - CURN	6	1
12	NLM3....A	NL MONEY SUPPLY - M3 - CURN	6	1
13	EMECBM1.B	EM MONEY SUPPLY: M1 - CURA	6	0
14	EMM2...B	EM MONEY SUPPLY: M2 - CURA	6	0
15	EMECBM3.B	EM MONEY SUPPLY: M3 - CURA	6	0
16	NLIMPGDSA	NL IMPORTS - CIF - CURN	5	1
17	NLEXPGDSA	NL EXPORTS - FOB - CURN	5	1
18	FRIMPGDSB	FR IMPORTS FOB - CURA	5	1
19	PREXPGDSB	FR EXPORTS FOB - CURA	5	1
20	ESOXT003b	ES ITS EXPORTS F.O.B. TOTAL - CURA	5	1
21	ESOXT009b	ES ITS IMPORTS C.I.F. TOTAL - CURA	5	1
22	ESEXPGDSD	ES EXPORTS - CONA	5	1
23	ESIMPGDSD	ES IMPORTS - CONA	5	1
24	ESEXPPRCF	ES EXPORT UNIT VALUE INDEX - NADJ	5	1
25	ESIMPPRCF	ES IMPORT UNIT VALUE INDEX - NADJ	5	1
26	BDEXPGDSB	BD EXPORTS OF GOODS (FOB) - CURA	5	1
27	BDIMPGDSB	BD IMPORTS OF GOODS (CIF) - CURA	5	1
28	BDEXPPRCF	BD EXPORT PRICE INDEX - NADJ	7	1
29	BDIMPPRCF	BD IMPORT PRICE INDEX - NADJ	7	1
30	ITEXPPRCF	IT EXPORT UNIT VALUE INDEX - NADJ	7	1
31	BDOCC011	BD REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
32	BGOCC011	BG REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
33	ESOCC011	ES REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
34	FNOCC011	FN REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
35	FRGCC011	FR REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
36	GROCC011	GR REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
37	IROCC011	IR REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
38	ITGCC011	IT REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
39	NLOCC011	NL REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
40	OEOCC011	OE REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
41	PTOCC011	PT REAL EFFECTIVE EXCHANGE RATES - CPI BASED - NADJ	5	0
42	BDESPPINF	BD PPI: MIG - NON-DURABLE CONSUMER GOODS - NADJ	7	0
43	BDPROPRCF	BD PPI: INDL. PRODUCTS, TOTAL, SOLD ON THE DOMESTIC MARKET -NADJ	7	0
44	BDESPIIEF	BD PPI: MIG - ENERGY - NADJ	7	0
45	FRESPPITF	FR PPI: MIG - INTERMEDIATE GOODS - NADJ	7	0
46	ITESPPINF	IT PPI: MIG - NON-DURABLE CONSUMER GOODS -NADJ	7	0
47	ITESPIIEF	IT PPI: MIG - ENERGY - NADJ	7	0
48	ESESPPITF	ES PPI: MIG - INTERMEDIATE GOODS - NADJ	7	0
49	ESESPPINF	ES PPI: MIG - NON-DURABLE CONSUMER GOODS - NADJ	7	0

Fabio Della Marra

50	ESPPDCNSF	ES PPI - CONSUMER GOODS, DURABLES - NADJ	7	0
51	ESPPINVSF	ES PPI - CAPITAL GOODS - NADJ	7	0
52	ESESPPIEF	ES PPI: MIG - ENERGY - NADJ	7	0
53	ESPROPRCF	ES PPI - NADJ	7	1
54	BGESPPITF	BG PPI: MIG - INTERMEDIATE GOODS - NADJ	7	0
55	BGESPPINF	BG PPI: MIG - NON-DURABLE CONSUMER GOODS - NADJ	7	1
56	BGESPPIIF	BG PPI: INDUSTRY - NADJ	7	0
57	NLESPPIITF	NL PPI: MIG - INTERMEDIATE GOODS - NADJ	7	0
58	EKPROPRCF	EK PPI: INDUSTRY - NADJ	7	0
59	ITCPWORKF	IT CPI EXCLUDING TOBACCO (FOI) - NADJ	7	0
60	ITCP7500F	IT CPI (1975=100) - NADJ	7	0
61	ITRAWPRCF	IT RAW MATERIALS PRICE INDEX - NADJ	7	0
62	ITPROPRCF	IT PPI - NADJ	7	0
63	FRCONPRAF	FR CPI (LINKED & REBASED) - NADJ	7	0
64	FRAGPRC.F	FR AGRICULTURAL PRICE INDEX - NADJ	7	0
65	FRAGIGSF	FR AGRICULTURAL INPUT PRICES - INVESTMENT GOODS & SERVICES - NADJ	7	1
66	BDCP7500F	BD CPI (1975=100) - NADJ	7	1
67	ESCONPRCF	ES CPI - NADJ	7	0
68	NLCONPRCF	NL CPI - NADJ	7	0
69	EMCONPRCF	EM CPI - NADJ	7	0
70	BDL.RELF	BD REAL EFFECTIVE FX RATE (REER) BASED ON UNIT LABOUR COSTS - NADJ	5	0
71	BDMWAGINF	BD WAGE&SALARY LEVEL,MTHLY BASIS - PRDG.SECT.(PAN BD M0191) NADJ	5	1
72	ESWAGES.F	ES WAGES: INCOME INDICATOR - VOLN	5	1
73	ESWAGES%F	ES WAGES: INCOME INDICATOR (%YOY) - VOLN	5	0
74	FRL.RELF	FR REAL EFFECTIVE FX RATE (REER) BASED ON UNIT LABOUR COSTS - NADJ	5	0
75	ITL.RELF	IT REAL EFFECTIVE FX RATE (REER) BASED ON UNIT LABOUR COSTS - NADJ	5	1
76	ITWAGES.F	IT CONTRACTUAL HOURLY WAGE: ALL WORKERS - NADJ	5	1
77	ITOLC007H	IT HOURLY WAGE RATE: INDUSTRY INCL. CONSTRUCTION - PROXY NADJ	5	1
78	ITWAGES%F	IT CONTRACTUAL HOURLY WAGE: ALL WORKERS (%YOY) - NADJ	5	0
79	NLL.RELF	NL REAL EFFECTIVE FX RATE (REER) BASED ON UNIT LABOUR COSTS - NADJ	5	0
80	NLOLC007H	NL HOURLY WAGE RATE: MFG - PROXY NADJ	5	1
81	BDIPTOT.G	BD INDUSTRIAL PRODUCTION INCLUDING CONSTRUCTION (CAL ADJ) - VOLA	5	0
82	BDESPISDH	BD IPI: MIG - DURABLE CONSUMER GOODS, VOLUME IOP (WDA) - VOLN	5	1
83	BDESPIESH	BD IPI: MIG-CAPITAL GOODS, VOLUME INDEX OF PRODUCTION (WDA) - VOLN	5	1
84	BDESPISNH	BD IPI: MIG - NON-DURABLE CONSUMER GOODS, VOLUME IOP (WDA) - VOLN	5	1
85	ESIPINTGH	ES INDUSTRIAL PRODUCTION - INTERMEDIATE GOODS - VOLN	5	1
86	ESIPINVSH	ES INDUSTRIAL PRODUCTION - CAPITAL GOODS - VOLN	5	1
87	ESESBASG	ES IPI: MANUFACTURE OF BASIC METALS, VOLUME IOP (WDA) - VOLA	5	0
88	ESIPOMNPH	ES INDUSTRIAL PRODUCTION - OTHER NON-METAL MINERAL PRODUCTS - VOLN	5	1
89	ESIPTOT.G	ES INDUSTRIAL PRODUCTION (WDA) - VOLA	5	0
90	ITIPTOT.G	IT INDUSTRIAL PRODUCTION - VOLA	5	0
91	NLIPTOT.G	NL INDUSTRIAL PRODUCTION EXCLUDING CONSTRUCTION - VOLA	5	0
92	FRIPTOT.G	FR INDUSTRIAL PRODUCTION - VOLA	5	0
93	EU18	EK PRODUCTION - TOTAL INDUSTRY EXCL. CONSTRUCTION - VOLA	5	0
94	BDNEWORDE	BD MANUFACTURING ORDERS - SADJ	5	0
95	BDRVNCARP	BD NEW PASSENGER CAR REGISTRATIONS - VOLN	5	1
96	BGACECARP	BG NEW PASSENGER CAR REGISTRATIONS - VOLN	5	1
97	ESCAR...O	ES REGISTRATIONS: PASSENGER CAR - VOLA	5	0
98	FRCARREGO	FR NEW CAR REGISTRATIONS (CAL ADJ) -VOLA	5	0
99	FRHCONMFD	FR HOUSEHOLD CONSUMPTION - MANUFACTURED GOODS - CONA	5	0
100	FRHCONDGD	FR HOUSEHOLD CONSUMPTION - DURABLE GOODS - CONA	5	0
101	ITNEWORDF	IT NEW ORDERS - NADJ	5	1
102	ITRETTOTF	IT RETAIL SALES - NADJ	5	1
103	BDCNFCONQ	BD CONSUMER CONFIDENCE INDICATOR - GERMANY - SADJ	2	0
104	BGCNFCONQ	BG BNB CONS. SVY.: CONSUMER CONFIDENCE INDICATOR (EP) - SADJ	2	0
105	BGCNFBUSQ	BG BUSINESS INDICATOR SURVEY - ECONOMY - SADJ	2	0
106	BGEUSIOBQ	BG IND.: OVERALL - ORD BOOKS - SADJ	2	0
107	BG000183Q	BG BNB BUS. SVY. - MANUFACTURING - NOT SMOOTHED - SADJ	2	0
108	BG000186Q	BG BNB BUS. SVY. - BUILDING - NOT SMOOTHED - SADJ	2	0
109	BG000189Q	BG BNB BUS. SVY. - TRADE - NOT SMOOTHED - SADJ	2	0
110	BGSURECSQ	BG BNB CONS.SVY.: ECON.SITUATION. FCST. OVER NEXT 12 MONTHS - SADJ	2	0
111	BGSURPUHQ	BG BNB CONS.SVY.: MAJOR HH.PURCH.FCST.OVER NEXT 12 MONTHS(EP)	2	0
112	ESINT384R	ES PRODUCTION LEVEL - INDUSTRY - NADJ	2	0
113	FRINDSYNQ	FR SURVEY: MANUFACTURING - SYNTHETIC BUSINESS INDICATOR - SADJ	2	0
114	FRSURMPMPQ	FR SURVEY: MANUFACTURING OUTPUT - RECENT OUTPUT TREND - SADJ	2	0
115	FRSURGMPQ	FR SURVEY: MANUFACTURING OUTPUT - ORDER BOOK & DEMAND - SADJ	2	0
116	FRSURGPDQ	FR SURVEY: MANUFACTURING OUTPUT LEVEL - GENERAL OUTLOOK - SADJ	2	0
117	FRSURTMPQ	FR SURVEY: MANUFACTURING OUTPUT - PERSONAL OUTLOOK - SADJ	2	0
118	ITHHFCESR	IT HOUSEHOLD CONFIDENCE SURVEY: FUTURE FINANCIAL POSITION - NADJ	2	0
119	ITCNFCONQ	IT HOUSEHOLD CONFIDENCE INDEX - SADJ	5	0
120	NLCNFBUSQ	NL CBS MFG. SVY.: PRODUCER CONFIDENCE INDEX - SADJ	2	0
121	NLEUSPCPR	NL CONSUMER SURVEY: MAJOR PURCH.OVER NEXT 12 MONTHS-NETHERLANDS	2	0
122	EKCENFBUSQ	EK INDUSTRIAL CONFIDENCE INDICATOR - EA - SADJ	2	0
123	EMEUSCCIQ	EM CONSUMER CONFIDENCE INDICATOR - EA - SADJ	2	0
124	EKEUSIPAQ	EK INDUSTRY SURVEY: PRODUCTION EXPECTATIONS (EA) - SADJ	2	0
125	EKEUBCI.R	EK BUSINESS CLIMATE INDICATOR-COMMON FACTOR IN IND. (EA) - NADJ	2	0
126	EKEUSESIG	EK ECONOMIC SENTIMENT INDICATOR (EA18) - VOLA	5	0
127	EMGBOND.	EM GOVERNMENT BOND YIELD - 10 YEAR	2	0
128	EMECB2Y.	EM GOVERNMENT BOND YIELD - 2 YEAR	2	0
129	EMECB3Y.	EM GOVERNMENT BOND YIELD - 3 YEAR	2	0
130	EMECB5Y.	EM GOVERNMENT BOND YIELD - 5 YEAR	2	0
131	EMECB7Y.	EM GOVERNMENT BOND YIELD - 7 YEAR	2	0
132	BDESSFUB	BD HARMONISED GOVERNMENT 10-YEAR BOND YIELD	2	0
133	FRESSFUB	FR HARMONISED GOVERNMENT 10-YEAR BOND YIELD	2	0
134	ESESSFUB	ES HARMONISED GOVERNMENT 10-YEAR BOND YIELD	2	0
135	BGSESSFUB	BG HARMONISED GOVERNMENT 10-YEAR BOND YIELD	2	0
136	ITESSFUB	IT HARMONISED GOVERNMENT 10-YEAR BOND YIELD	2	0
137	ITINTER3	IT INTERBANK DEPOSIT RATE-AVERAGE ON 3-MONTHS DEPOSITS	2	0
138	MSEROP\$ E	MSCI EUROPE US\$ - PRICE INDEX	5	0
139	ITALGSI& E	ITALY-DS Inds Gds & Svs - PRICE INDEX	5	0
140	INDGSBD& E	GERMANY-DS Inds Gds & Svs - PRICE INDEX	5	0
141	INDGSFR& E	FRANCE-DS Inds Gds & Svs - PRICE INDEX	5	0

A forecasting performance comparison of dynamic factor models based on static and dynamic methods

142	INDUSBD E	GERMANY-DS Industrials - PRICE INDEX	5	0
143	INDUSFR E	FRANCE-DS Industrials - PRICE INDEX	5	0
144	INDUSIT E	ITALY-DS Industrials - PRICE INDEX	5	0
145	FINANFR E	FRANCE-DS Financials - PRICE INDEX	5	0
146	FINANBD E	GERMANY-DS Financials - PRICE INDEX	5	0
147	FINANIT E	ITALY-DS Financials - PRICE INDEX	5	0
148	CNSMGFR E	FRANCE-DS Consumer Gds - PRICE INDEX	5	0
149	CNSMGBD E	GERMANY-DS Consumer Gds - PRICE INDEX	5	0
150	CNSMGIT E	ITALY-DS Consumer Gds - PRICE INDEX	5	0
151	OILGSEM E	EMU-DS Oil & Gas - PRICE INDEX	5	0
152	BMATREM E	EMU-DS Basic Mats - PRICE INDEX	5	0
153	INDUSEM E	EMU-DS Industrials - PRICE INDEX	5	0
154	RITDVEM E	EMU-DS Divers. REITs - PRICE INDEX	5	0
155	CNSMGEM E	EMU-DS Consumer Gds - PRICE INDEX	5	0
156	HLTHCEM E	EMU-DS Health Care - PRICE INDEX	5	0
157	TELCMEM E	EMU-DS Telecom - PRICE INDEX	5	0
158	UTILSEM E	EMU-DS Utilities - PRICE INDEX	5	0
159	FINANEM E	EMU-DS Financials - PRICE INDEX	5	0
160	CNSMSEM E	EMU-DS Consumer Svs - PRICE INDEX	5	0
161	TECNOEM E	EMU-DS Technology - PRICE INDEX	5	0
162	EMSHRPRCF	EM DATASTREAM EURO SHARE PRICE INDEX (MONTHLY AVERAGE) - NADJ	5	0
163	BDMLM006Q	BD REGISTERED UNEMPLOYMENT: RATE (ALL PERSONS) - SADJ	2	0
164	BDMLM005O	BD REGISTERED UNEMPLOYMENT: LEVEL (ALL PERSONS) - VOLA	5	0
165	ITMLFT15O	IT HARMONISED UNEMPLOYMENT: LEVEL, ALL PERSONS (ALL AGES) - VOLA	5	0
166	ITMLRT16Q	IT HARMONISED UNEMPLOYMENT: RATE, ALL PERSONS (ALL AGES) - SADJ	2	0
167	ITMLRT14Q	IT HARMONISED UNEMPLOYMENT: RATE, ALL PERSONS (AGES 15-24) - SADJ	2	0
168	ITMLRF16Q	IT HARMONISED UNEMPLOYMENT: RATE, FEMMES (ALL AGES) - SADJ	2	0
169	ITMLRM16Q	IT HARMONISED UNEMPLOYMENT: RATE, HOMMES (ALL AGES) - SADJ	2	0
170	PRESTUNFO	FR UNEMPLOYMENT: TOTAL - TOTAL - VOLA	5	0
171	FRMLRT14Q	FR HARMONISED UNEMPLOYMENT: RATE, ALL PERSONS (AGES 15-24) - SADJ	2	0
172	FRMLRT15Q	FR HARMONISED UNEMPLMT.: RATE, ALL PERSONS (AGES 25 AND OVER) - SADJ	2	0
173	FRMLRT16Q	FR HARMONISED UNEMPLOYMENT: RATE, ALL PERSONS (ALL AGES) - SADJ	2	0
174	FRMLRF16Q	FR HARMONISED UNEMPLOYMENT: RATE, FEMMES (ALL AGES) - SADJ	2	0
175	FRMLRm16Q	FR HARMONISED UNEMPLOYMENT: RATE, HOMMES (ALL AGES) - SADJ	2	0
176	ESMLM005O	ES HARMONISED UNEMPLOYMENT: LEVEL, ALL PERSONS (ALL AGES) - VOLA	5	0

The Transformation Codes, Tcode, in table A.1, are defined as reported below:

$$(16) \quad z_{it} = x_{it}\delta(T_{code} - 1) + [(1 - L)x_{it}]\delta(T_{code} - 2) + [(1 - L)^2x_{it}]\delta(T_{code} - 3) + \ln(x_{it})(T_{code} - 4) + (1 - L)\ln(x_{it})\delta(T_{code} - 5) + (1 - L)^2\ln(x_{it})\delta(T_{code} - 6) + (1 - L)(1 - L^2)\ln(x_{it})\delta(T_{code} - 7)$$

where x_{it} represents the raw time series x_i at time t and $\delta(\cdot)$ the Dirac delta function.

REFERENCES

1. J. H. Stock and M. W. Watson, Forecasting using principal components from a large number of predictors, *Journal of the American Statistical Association*, vol. 97, no. 460, pp. 1167–1179, 2002.
2. J. H. Stock and M. W. Watson, Macroeconomic forecasting using diffusion indexes, *Journal of Business & Economic Statistics*, vol. 20, no. 2, pp. 147–162, 2002.
3. M. Forni, M. Hallin, M. Lippi, and L. Reichlin, The generalized dynamic factor model: Identification and estimation, *Review of Economics and Statistics*, vol. 82, no. 4, pp. 540 – 554, 2000.
4. M. Forni, M. Hallin, M. Lippi, and L. Reichlin, The generalized dynamic factor model: One-sided estimation and forecasting, *Journal of*

- the American Statistical Association*, vol. 100, pp. 830 – 840, 2005.
5. M. Forni, M. Hallin, M. Lippi, and P. Zaffaroni, Dynamic factor model with infinite dimensional factor space: representation, *Journal of Econometrics*, vol. 185, pp. 359–371, 2015.
 6. M. Forni, M. Hallin, M. Lippi, and P. Zaffaroni, Dynamic factor model with infinite dimensional factor space: Asymptotic analysis, tech. rep., EIEF Working Paper 2016/07, Einaudi Institute for Economics and Finance, 2016.
 7. M. Forni, A. Giovannelli, M. Lippi, and S. Soccorsi, Dynamic factor model with infinite dimensional factor space: forecasting, tech. rep., CEPR Discussion Paper No. DP11161, 2016.
 8. J. Boivin and S. Ng, Understanding and comparing factor-based forecasts, *International Journal of Central Banking*, vol. 1, no. 3, pp. 117 – 151, 2005.
 9. A. D’Agostino and D. Giannone, Comparing alternative predictors based on large-panel factor models, *Oxford bulletin of economics and statistics*, vol. 74, no. 2, pp. 306–326, 2012.
 10. C. Schumacher, Forecasting german gdp using alternative factor models based on large datasets, *Journal of Forecasting*, vol. 26, no. 4, pp. 271–302, 2007.
 11. A. H. den Reijer *et al.*, Forecasting dutch gdp using large scale factor models, tech. rep., Netherlands Central Bank, Research Department, 2005.
 12. M. Forni and M. Lippi, The generalized dynamic factor model: representation theory, *Econometric theory*, vol. 17, no. 06, pp. 1113–1141, 2001.
 13. M. Hallin and M. Lippi, Factor models in high-dimensional time series—a time-domain approach, *Stochastic Processes and their Applications*, vol. 123, no. 7, pp. 2678–2695, 2013.
 14. M. Forni, D. Giannone, M. Lippi, and L. Reichlin, Opening the black box: Structural factor models with large cross sections, *Econometric Theory*, vol. 25, no. 05, pp. 1319–1347, 2009.
 15. R. Giacomini and H. White, Tests of conditional predictive ability, *Econometrica*, vol. 74, no. 6, pp. 1545–1578, 2006.
 16. R. Giacomini and B. Rossi, Forecast comparisons in unstable environments, *Journal of Applied Econometrics*, vol. 25, no. 4, pp. 595–620, 2010.